Quantitative Practices

Peter J. Denning
George Mason University

11/24/96

It is not literacy but practices that create actions and constitute expertise. Literacy deals with descriptions, practices with data, design, uncertainty, trade-offs. Although schools stress literacy, many important practices defy description.

Can citizens distinguish the dross from the essential in the musings of technology experts? Can they make sense of newspaper commentaries? Can they understand a risk assessment or tell if it is reasonable? Although not needing level of quantitative expertise of a scientist or engineer, the average citizen does need to be able to cope with such questions every day.

For three decades I have been an engineer working with computing and information technology. Quantitative methods are used extensively in my own field. I have observed them in all the other engineering disciplines, in the sciences, and in many other fields. They are practiced everywhere.

Yet I am concerned that the current discussion, focused on literacy rather than practice, may not lead to the educational outcome it seeks. In my view, the central question is, “What quantitative practices does a person need to know to be effective?” A focus on literacy leads toward descriptions and observations of practices, but not into the practices themselves. Literacy is like the menu in a restaurant; it tells you about the dinner, but it cannot feed you. The world of practices is messy: practices defy precise descriptions; new practices are constantly emerging; others are becoming obsolete; practices evolve in harmony with technologies. Despite the fuzziness and dynamics of practices, it is essential for us to understand their importance in what it means to be educated. Then we will be able to draw some new conclusions about “literacy” and about such apparently mundane questions as whether students should be allowed to bring calculators to exams.

A Short Story

A few years ago, my elder daughter came to me with a request to help her do her math homework; she was totally stuck with a set of word problems about proportions. (Example: "You measure the length of the shadow of a 6' vertical stick as 10'. You measure the length of the shadow of a tree as 100'. How tall is the tree?") She said she understood the concept of proportions but couldn't see how to use it for the word problems. I asked her to explain the concept of proportions. She said: "You're given that A:B is the same as C:D. The word problem gives you three of the four variables A, B, C, and D. You plug in the values and solve for the missing value." Sounded impressive. But she was utterly unable to connect this with the word problems. She did not understand how to represent entities in the word problem with symbols A, B, C, or D. I asked her to draw a diagram of the situation behind the word problem. She could not do it. She had the same difficulty with the other word problems. She had no conception of how to assign variables or to draw a picture of the situation.

So I said, "Look, I'm going to solve the first five of these problems. I'm going to think out loud and draw pictures. You just watch me do it. Don't try and figure anything out, just watch." By the time I'd finished the fifth problem she said, "I think I see what is going on." And she then went on to solve the other five problems, each one with progressively less assistance from me.

After questioning her about all her mathematics classes, I concluded that she had never seen a mathematician in action, solving problems. She had never witnessed the *practice* of mathematics. She had never been involved in the practices of assigning variables or drawing pictures. Without these practices, the theory and principles were useless to her. She could not act effectively. She had been shortchanged by her curriculum, which could not deliver what it promised.

I concluded from conversations with others that my daughter was not alone. Many young people cannot practice mathematics after finishing the high-school math curriculum. We call that "functional illiteracy" or sometimes "innumeracy" [4]. High School Mathematics a beautiful curriculum that organizes the principles in a very logical progression. But it does not teach the practice of mathematics. It is as if the designers of the curriculum were stuck in the notion that practice is the application of theory and will follow naturally when a person is well-grounded in theory.

Mathematics is more than that. It is a language, a discourse, and a set of practices. If you don't get a chance to observe a mathematician at work or work with a mathematician, you won't learn mathematical practices, and it may not occur to you that mathematicians do anything of value. Every time I ask someone to describe how they learned something they seem to do well, they always recall a moment at the beginning when they observed people doing the thing and producing useful results, and they recall later moments when they got involved in doing it themselves; in fact, the practice kindled their interest in learning the theory.

The Importance of Practices

My daughter's story recalls an important distinction between theory and practice. To explore this distinction more deeply, I would like to replace "theory" with the broader term "descriptions." Descriptions are the theories, representations, models, data, facts, rules, and narratives of a domain. A practice is a habitual pattern of action engaged in routinely by people in a domain, usually without thought; practices include the standard patterns, routines, procedures, processes, and habits of people acting in the domain. Mathematicians and journalists operate primarily with descriptions. Managers, sports professionals, and coaches operate primarily with practices. Engineers, scientists, doctors, and lawyers deal with both.

Descriptions and practices overlap, but neither contains the other. Think of the difference between the sports journalist and the basketball player, between the financial analyst and investor, between the professor of engineering and the licensed professional engineer, or between the menu and the dinner. The journalist can tell us why the ball-player shoots well, but cannot himself shoot; ball-players are notorious for their inability to describe what they do in ways that help others imitate them. The financial analyst tells us why the stock market is rising or falling but is quiet about his dismal record as an investor. A fluid dynamicist describes in detail the method of calculating Euler flows around a wing, but cannot get the algorithm to run fast on a Connection Machine.

We educators incline toward the domain of descriptions. It's our stock in trade, the stuff of lectures and presentations. It's where we place all the models and theories of the world that we want our students to learn. We contrast education and training. We locate education in the more familiar territory of descriptions; we harbor suspicions of training, which is about imparting specific practices. We have been brought up on the theory that action happens when we apply a (mental) model of the world to the situation at hand. Descriptions seem rational; practices do not.

Quantitative Literacy concerns a student's familiarity with numbers and numerical manipulations. The term "literacy" already reveals a bias toward descriptions. My purpose in the remainder of this essay is to suggest that there is a great richness in practices of working with data and numbers, practices that are not well captured as descriptions. I suggest that we should examine "quantitative practices" rather than "quantitative literacy" to find the answers to our questions about what to teach our students (see Figure 1).

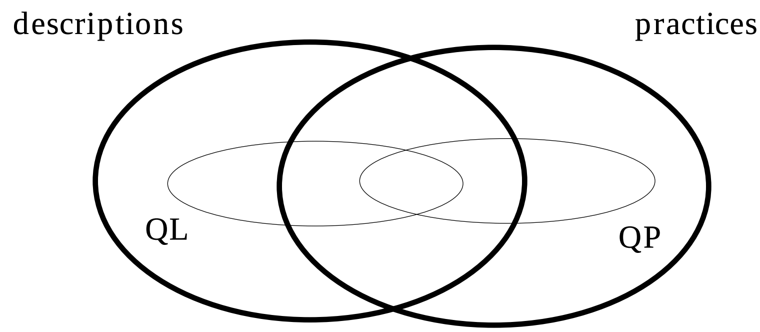

Figure 1. Descriptions are the representations, rules, facts, data, facts, theories, models, and narratives of phenomena in a domain. Practices are the standard patterns, routines, procedures, processes, and habits of people in the domain. These domains overlap, but neither contains the other. Quantitative practices (QP) emphasize measurement, evaluation, model validation, and design trade-offs. Quantitative Literacy (QL) emphasizes familiarity with descriptions and basic practices. The overlap of QP and QL are the practices for which we have precise descriptions. Many disciplines such as engineering, science, medicine, and economics rely heavily on quantitative practices. QP rather than QL should be the target of our educational objectives.

There are important quantitative practices for which we have no effective description. By effective, I mean that someone else could take the description, understand it, practice it, and finally appropriate the practice for himself or herself. Here are a few examples of important practices in engineering and science for which no effective descriptions are known:

Fractal art
Formulating a null hypothesis
Designing experiments
Collecting data
Knowing when you have enough data
Finding approximations for very large data sets
Constructing effective reports
Constructing and validating a model
Finding patterns in data
Classifying data into clusters
Estimating and removing measurement errors

Human factors experiments
Designing and validating a heuristic algorithm
Evaluating trade-offs in designing a system

This list can easily be extended to include other domains; for example,

Determining whether a Fed interest rate hike will induce recession
Modeling a city (e.g., the game SimCity)
Predicting traffic jams during rush hour
Finding stock market cycles
Estimating when the river will crest after a flooding rain
Calculating the sag of a bridge during rush hour
Placing fiber optics links to relieve Internet traffic jams
Routing airplanes among cities and maintenance stops
Determining the throughput of an assembly line
Locating oil fields from test drillings
Projecting the financial position of a company for the next two years
Measuring chemical levels in a blood sample
Finding the pollen count

To the best of my knowledge, most approaches to quantitative literacy have assumed implicitly that the quantitative practices with which students must be familiar have effective descriptions. For the reasons stated above, I believe this assumption is misleading at best and invalid at worst.

Tools

Practices are usually supported by tools and in many cases are impossible without the tools. Typing, a central practice in using computers, is an example. Its essential tool is the keyboard: you cannot type without a keyboard. And not just any keyboard: you will have difficulty with a Dvorak keyboard if you were trained on a Qwerty keyboard. As a physical object, a keyboard has no meaning to someone without the practice of typing: it would appear as an utter mystery to a time traveler from the Middle Ages. You are unlikely to succeed at using a computer today without knowing how to type: as you learn computers you cross-appropriate typing from another domain.

Practices and tools live in a symbiotic relationship: each needs the other, and neither is meaningful without the other. Tools evolve in harmony with practices. The desire for better, faster, cheaper results stimulates technologies to enable practices to be more effective. Modern banking, budgeting, and financial reporting are impossible without the spreadsheet; accounting and financial reporting have made enormous advances since the spreadsheet was invented in the 1980s. Yet a financial spreadsheet is meaningless to someone who has never seen a list of accounts.

Tools enable practices, but do not confer them. People learn practices from other people. The hand calculator and spreadsheet, which did not exist half a century ago, are essential tools for modern quantitative practice.

Competence

Think of a highly competent person, one whom you might describe as a virtuoso or even a master. What do we mean when we say this person is competent in a field? We mean that that person understands the history, methods, practices, boundaries, current problems, and relationships to other fields. More than that, we mean that the person can *perform* effectively in that field. The actions of the highly competent person impress us with their skill and finesse.

The important observation is that we associate competence, effectiveness, and knowledge with actions. We do no expect to be able to perform like the experts simply from listening to their descriptions of their actions or mental processes. Indeed, it is quite normal that the highly competent person is unable to give a clear description of how he does it.

Since the 1960s, the philosopher Hubert Dreyfus has investigated human competence, inspired by the question of whether expert systems (and other machines intended to mimic human behavior) can in principle become as competent as human experts. He identified six competence levels, corresponding to ever-higher demonstrated capabilities for performance: beginner, rookie, professional, expert, virtuoso, and master. Dreyfus demonstrated that it takes time for a person to acquire the new skills required for each higher level, and that the highest levels may take many years of practice to attain. His surprising conclusion is that that rule-following behavior is not present at the expert, virtuoso, and master levels. Consequently, expert systems could never become “expert” [3]. In fact systems based on rules can hardly be expected to attain the competence level of a professional. In so doing, Dreyfus challenged the conventional wisdom that expert behavior can be described formally with enough precision that a machine could do it or that someone else could learn the same behavior from the description. His claims have stood the test of time.

Dreyfus’s conclusion is important in the present discussion. Many important practices are not the “application” of rules. They cannot be learned from descriptions. To be “literate” is to be versed in descriptions. The only way to learn the necessary practices is to do them.

Obsolescence

Some people say that many high school graduates are (quantitatively) illiterate because they cannot “do arithmetic”, meaning that they cannot do routine arithmetic calculations by hand, with pencil and paper. These critics say that calculators are crutches that should not be allowed for exams. I find this whole discussion rather muddled. It does not identify the domain (context) in which the calculations are important or the criteria of effectiveness. Where calculations are important, effectiveness is usually correlated with the number of error-free calculations completed. Therefore a person skilled with a calculator will be more effective than a person who does the arithmetic manually, and a person skilled with a spreadsheet will be more effective than the one with the calculator. Even the mundane business of purchasing groceries for a large family with a tight budget can benefit from a shopper who can do arithmetic with a calculator. The important thing is that the person embody effective practices and have the tools (such as calculators or spreadsheets) to support those practices.

When a practice is no longer effective, we say that it (and its tools) have become obsolescent. Thirty years ago, the practices of slide-rule calculations and drafting were central to engineering. Today the calculator and spreadsheet are central and the slide rule is an historical curiosity; the CAD program has pushed drafting into the dustbins of history. The word processor has made the typewriter obsolete, and the speech-recognizer is likely to make the practice of typing at keyboard obsolete within the next generation. The practice of writing letters and business memos is being replaced by electronic mail and Internet communication. The important point is that as some practices become obsolete, they are replaced by new practices that make people more effective. The degree of skill expected of the practitioner goes up. The new practices are supported by more sophisticated technology.

Should we worry about students relying so much on calculators that they cannot do their banking or buy their groceries without taking a calculator along? I think not. We should worry instead about students who are not facile with the calculator. Modern shoppers shop for balanced meals within fixed budgets --- a more sophisticated practice than arithmetic calculation. Working adults must manage cash flows within budgets and properly report income on their tax forms --- also a more sophisticated set of practices than arithmetic calculation. The job of the shopper is greatly facilitated by the calculator, and of the working adult by the automated checkbook (e.g., Quicken). The person who is competent solely at arithmetic cannot perform at the same level as the person who is competent with the calculator or spreadsheet. And so it may well be that the skill of manual arithmetic will become obsolete.

Quantitative Practices in Practice

Many of the foregoing statements are general. I would like to illustrate them with a closer look at several disciplines, beginning with my own, computing. I do not intend to be exhaustive, but I want to illustrate well enough to drive home the point that quantitative practices are pervasive.

Computing

The discipline of computing has been defined as the body of knowledge about the automation of step-by-step procedures -- i.e., the set of phenomena surrounding computers. It is the discipline whose practitioners help other people take care of their concerns about representing, storing, retrieving, communicating, and processing data, and in coordinating their actions with each other through exchange of data. The subject matter of the discipline can be represented as a matrix depicting eleven major subareas and the three processes: theory, experimentation, and design [1,2].

	Theory	**Experimentation**	**Design**
Algorithms & Data Structures			
Programming Languages			
Architecture			
Numerical & Symbolic Computation			
Operating Systems and Networks			
Software Methodology & Engineering			
Databases & Information Retrieval			
Artificial Intelligence & Robotics			
Human Computer Communication			
Computational Science			
Organizational Informatics			

In each of the squares, you might imagine a detailed description of the kinds of problems addressed, the accomplishments, and the open questions. In every case, the processes of experimentation and design rely extensively on quantitative practices. Let me give some examples.

- When selecting an algorithm, a designer needs to know how fast the algorithm will be or how much storage it will require; although many of these questions can be answered mathematically, it is increasingly common to answer them with experiments and simulations, especially when the algorithm relies on approximations or heuristics.
- It might seem that designing a computer microchip is mainly an exercise in computer-aided design and logic simulation. In fact, it has turned into a highly quantitative exercise. Designers hesitate to place an instruction in the computer's instruction set unless they can demonstrate that real programs will run faster. Assessing this requires detailed statistical analyses of frequencies of instructions in actual programs.
- Programs such as Mathematica, Maple, or Reduce that manipulate, evaluate, and display mathematical expressions rest heavily on quantitative methods, especially in determining the error of a computation and in computing the graphs of a function.
- The Internet is a complex web of interconnected computers and network protocols. Network engineers are constantly measuring network traffic, managing routing, and reconfiguring line capacities to minimize traffic delays and response times. They use sophisticated heuristic algorithms to find near-optimal line capacities. Capacity planners frequently use queueing models to calculate throughput and response times of local computer systems attached to the network.

- Software engineers subject their programs to rigorous tests, seeking to determine if the proper internal control paths are followed for each possible pattern of input data.
- The algorithms used to search very large data bases and to correlate values between data bases are heuristic, and can be validated only by extensive experimentation and testing.
- Designers of learning machines make heavy use of heuristic algorithms for everything from searching for the best next move in a chess game to the Turing test itself ("for how long can a machine fool a human interrogator?").
- Designers of graphical interfaces resort to human factors measurements to assess how well a design feature or display method works in practice.
- Scientific programmers nearly always begin with a mathematical model of the physical phenomenon they wish to study, and then construct software programs for supercomputers to evaluate and display those models. Since the models almost always contain approximations, these programmers must also validate their models against real data.

Other Fields

Engineering. Civil engineers carry out surveys, design experiments to minimize errors in surveying instruments, test structural plans against computer-based models, calculate quantities of materials needed, and estimate costs of construction. Electrical engineers simulate logic circuits, analyze instruction frequencies in program codes, experiment with cache sizes on microchips, analyze buses for contention among processors connected to them, analyze communication channels for errors, measure and reconfigure networks for maximal information flows, and assess reliability of power and telephone grids. Mechanical engineers build models to compute dynamic and static stresses in structures, estimate throughput and response time of manufacturing lines, build models that are incorporated into feedback control systems, calibrate instruments, machine to ever-finer tolerances, and compute the lifts, drags, and turbulences affecting aircraft. Chemical engineers estimate flow and reaction rates in petroleum plants, calculate optimal yields of chemical production processes, compute the properties of materials (such as heat shields) in inaccessible places (such as Jupiter's atmosphere), compile detailed assays of the chemical composition of unknown substances. Petroleum engineers estimate the yields of oil fields from soils dug up from drill-holes.

Astronomy. Astronomers use sophisticated algorithms to detect very faint objects in the visual fields of telescopes. They search for evanescent signals from extra terrestrial intelligences. They calculate whether split images of distant galaxies might be caused by an intervening black-hole lens. They model the evolution of the universe and gather data to support or reject hypotheses about the birth and death of the universe. They track local objects such as comets and asteroids and alert the public to their positions.

Environment. Atmospheric scientists monitor rainfall, project periods of drought, track the status of the Ozone Hole, and compute pollution alerts for urban centers. Meteorologists forecast weather conditions based on current measurements. Oceanographers measure ocean currents and temperatures and use them to project fish movements and conditions (such as El Niño) that will affect weather. Geologists predict earthquake probabilities and calculate flows of toxins through underground waterways. Global climate modelers project long range weather conditions by combining these models.

Bioinformatics. Genetic engineers use computer models to calculate which DNA sequences are most likely to endow an organism with a desired property. They conduct statistical analyses and cross-correlations of DNA sequences recorded in very large databases. Epidemiologists use computer models to estimate the spread of diseases. AIDS researchers use computer models to estimate the most likely mutations of the HIV in preparation for designing drugs and vaccines.

Medicine. Biomedical engineers seek out and then incorporate rules of thumb, heuristics, and medical guidelines into "intelligent machines" that make good diagnoses. Medical researchers conduct statistical analyses and correlations of medical data, looking for confirmation of hypotheses about the causes or inhibitors of disease. They perform controlled experiments on new drugs or proposed medical procedures. Lab technicians measure blood chemical compositions. Instrument builders construct noninvasive methods of measuring various conditions in the body.

Finance. Financial advisors build spreadsheet models of an individual's assets incomes and expenses to devise plans for attaining wealth targets for retirement. Accountants compile cost and revenue projections of a company for several years in advance, and analyze sensitivity of results to assumptions used in the forecast. Bankers monitor and calculate cash reserves and the present values of future investments and liabilities.

Economics. Economists build models to enable forecasting national and global economic systems and determining the possible effects of public policies on growth or shortages. They analyze stock market data to look for trends that would interest investors. They calculate the optimal interest rates and money supplies to control inflation.

Management. Management scientists study which reporting methods are most effective. They map out organizational coordination processes, measure them, and project whether proposed reorganizations will help. They build models of their organizations as large feedback systems that can be evaluated to project the long term effects of current policies.

Law. Lawyers conduct extensive database searches for court rulings that might set precedent for a current case; these searches often include statistical analyses of the results. They are helping to build software programs that do routine legal tasks (such as wills, deeds, and powers of attorney) on home computers.

Literature. Literacy scholars analyze texts for the frequency of occurrence of letters, words, and phrases; they use the results for everything from tracking evolving meanings to testing whether a given person might have authored a document (e.g., were the works of Shakespeare really written by Shakespeare? Was Joe Klein really the anonymous author of a political book?). They can then catalogue guidelines for style and related words into style checkers and automated thesauruses for desktop computers.

Implications

Quantitative practices pervade a wide variety of fields. The practices show up in the ways people deal with data, measurements, instruments, experiments, evaluations, models, predictions, forecasts, and trade-offs. Quantitative questions arise even in fields such as law, literature, and medicine, which traditionally have not been regarded as quantitative disciplines. The computer creates a rich variety of opportunities for people in these disciplines to import quantitative methods. Students of the traditionally non-quantitative disciplines are finding themselves engaged increasingly in quantitative practices in their daily routines. It gives them a competitive edge.

Students of the traditionally quantitative disciplines (e.g., science, engineering, mathematics, statistics, and computing) must master the quantitative practices that are so important for their daily routines. Much of the university curriculum in these disciplines is organized to help students learn these practices. For this reason, the faculty are concerned that entering students come with basic quantitative practices. These include numeracy, working knowledge of algebra and calculus, and some exposure to statistics -- the practices of gathering and recording data, monitoring errors in measurements, and extrapolating trends.

The foregoing analysis strongly suggests that we need to look differently at the role of quantitative literacy in education. We need to reframe the question, focusing not on quantitative literacy but on quantitative practices. Much of what looks like “functional illiteracy” is in fact an absence of relevant practices. Curriculum changes intended to eliminate “functional illiteracy” should get students involved in the practices; merely offering better descriptions will not help.

Quantitative practices deal a lot with numbers, uncertainty, errors in data, design of experiments, creation of models, validations, drawing conclusions from data, making trade-offs, and the like. They cannot be taught at blackboard. Teachers must engage their students with labs, field work, simulation games, and other means of involving them in the practices.

However, we should not confuse tools (e.g., calculators, computers) with practices. Tools enable and support practices; but it is people who have the practices and teach them to other people. Giving tools to students, and even showing them how to use them, is not sufficient to teach practices. Involvement with the practices is the only way.

We must not forget that what constitutes an effective practice is domain-dependent. Practices are also time-dependent because what is effective today may be obsolete next year. To some educators, this lack of "timelessness" makes practices seem ephemeral and not a worthy part of a curriculum. But practices as a phenomenon are timeless, even if particular practices change over time.

In education and society, we need to grant practices an equal level of respect with descriptions. There are many important practices that do not have precise descriptions and cannot be learned by listening to and memorizing descriptions. This means we will have to give up our aversion to "training". Training -- the learning of important practices -- is part of education. The masters are skilled performers. Children cannot attain mastery by studying descriptions.

Engineering, sciences, mathematics, statistics, and computing are pervaded by quantitative practices. These disciplines cannot exist without them, and any young person aspiring to a technology profession will need to know quantitative practices. These days, with the help of the computer, practitioners of other disciplines and professions are finding that knowledge of quantitative methods gives them a competitive edge in their fields. Most non-technical citizens also need some quantitative practices to help them cope with life and work in a technological society, to make sense of the data they encounter, and to evaluate risks.

Quantitative practices are the dinners served at the educational table, the morsels described by the menus of quantitative literacy. For life and work, for citizenship and education, students need immersion in the messy world of practice as much as in the packaged world of literacy.

Readings

Denning, Peter. 1993. "The structure of Computer Science". *Encyclopedia of Computer Science* (A. Ralston, ed.). Reilly.

Denning, Peter, Douglas Comer, David Gries, Michael Mulder, Allen Tucker, Joseph Turner, and Paul Young. 1993. "Computing as a discipline." ACM *Communications 32*, 1 (January). 9-23.

Dreyfus, Hubert. 1992. *What Machines Till Can't Do.* MIT Press.

Paulos, John Allen. 1988. *Innumeracy*. Vintage Books.